Entry and social efficiency under Bertrand competition and asymmetric information

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Abstract

This paper explores the welfare implications of free entry when firms face known entry costs, but production costs are privately known. Upon entering, firms compete in prices to supply a homogeneous good. Our framework yields results that are more nuanced than those of the literature on free entry, where there is either insufficient or excessive entry. Depending on the distribution of costs, the value of the entry fee, and the number of potential entrants, it is possible to have both excessive and insufficient entry as parameters change. We also show that the existence of entry costs fundamentally changes one of the key results of Spulber (1995) on the convergence of the equilibrium price to the competitive equilibrium. Instead, with entry costs, we have shown that the probability of excessive entry goes to one as the number of potential firms goes to infinity.

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1 Introduction

The last decade saw a number of businesses emerge that use technology to disrupt established markets. Examples include Uber and Airbnb, which have a number of features in common, in addition to the use of a digital platform.[1]

Two features are of particular interest. First, there is a clear distinction between the product development phase and the decision to enter a market; firms may have already invested in R&D and created a new product prior to making a decision to enter a market. Firms, however, face entry costs to bring the new product to the market. The entry costs may take the form of legal and regulatory hurdles that businesses have to overcome to commence operations.

The second feature of interest is that at the time firms make the decision to enter the market, they do not know how many competitors they will face once they enter. Other firms may also be seeking to overcome legal and regulatory hurdles to enter the same markets. Moreover, the business model of potential entrants, and therefore their cost structure, is also likely to be unknown, or not known for certain, when a decision to enter a market is made.

This paper studies entry and price decisions of firms that operate in markets that exhibit these two features. In particular, we examine the welfare implications of free entry when firms face known entry costs, but total (fixed and variable) production costs are privately known. Upon entering, firms compete in prices to supply a homogeneous good.

The focus on Bertrand competition under asymmetric information is well-justified. Beyond the trivial observation that firms' costs are typically not observable by third parties, there is a fundamental inconsistency with Bertrand competition under full information. This inconsistency arises, for example, when firms have constant marginal costs and one firm has an absolute cost advantage over its rivals.

In this instance, the Bertrand equilibrium price is equal to second lowest marginal cost among all firms, and the lowest cost firm captures the entire market. However, as pointed out by Spulber (1995), in this equilibrium all other firms earn zero profit, and yet the equilibrium logic requires them to be ready to compete with the winning firm. In contrast, as Spulber (1995) shows, when firms' costs are privately known and there are no entry costs, all firms have positive expected profits at the equilibrium with asymmetric information, and therefore they have an incentive to enter the market

[1] See for example the info-graphics describes how Uber and Airbnb started available at, respectively, http://gulfelitemag.com/startup-bottom-uber-started/ and https://blog.adioma.com/how-airbnb-started-infographic/.

and engage in price competition.

In Spulber's model, firms only incur costs if they produce, and therefore the issue of entry is moot; entry is always efficient. In contrast, we consider the case where firms need to incur an entry cost to be able to compete in the market place. More specifically, we assume that firms make their entry decisions (i.e., their decisions to incur a sunk cost and enter) simultaneously, observe how many other firms have entered, and then choose a pricing strategy (as a function of their private information). We show that only low cost firms enter the market, in expectation of being able to successfully compete in the market. This is in contrast with the literature on entry in oligopoly market under full information, which assumes that firms' costs are identical, and therefore do not address the coordination issue of how to select the firms who enter and those that do not.[2]

The existence of entry costs yields results that are fundamentally different from Spulber (1995), both in terms of the characterization of the equilibrium and its properties. For example, in Spulber (1995), when the number of firms goes to infinity, the equilibrium price converges to the competitive equilibrium. However, we show that when firms face entry costs, only a subset of firms enter the market, leading to a higher equilibrium price than in Spulber (1995).

We also show the seminal result of Mankiw and Whinston (1986) that there is always a bias towards excessive entry is no longer true under asymmetric information and entry costs.[3] In the same vein, the result in Sharkey and Sibley (1993) that an increase in the number of potential competitors puts more probability weight on higher prices (in the symmetric mixed strategy equilibrium) is no longer necessarily true either. With asymmetric information and entry costs, there is a trade-off between the more cut-throat competition that arises from the entry of only low cost firms and the duplication of entry costs. This implies that it is possible to have either insufficient or excessive entry, depending on the nature of the distribution of costs, the number of potential firms and the entry costs.

There is a literature with regards to uncertainty about firms' characteristics in Bertrand model. For instance, Janssen and Rasmusen (2002) describe a model where

[2]For example, in their seminal paper, Mankiw and Whinston (1986) show that in homogeneous product market a business-stealing effect always creates a bias toward excessive entry: ignoring the integer constraint on the number of firms and the coordination issue identified above, marginal entry is more desirable to the entrant than it is to society because of the output reduction entry causes in other firms.

[3]There are other circumstances under which the excessive entry result does not hold. For example, Ghosh and Morita (2007) show that in presence of vertical integration free entry can lead to a socially insufficient number of firms.

some firms may be inactive with positive probability. They characterize mixed-strategy equilibrium and show in the Bertrand model the profit of firms increase in the probability that firms are inactive. However, they do not discuss entry and cost uncertainty as we do.

Finally, this paper is also related to the literature on auctions with entry (e.g., McAfee and McMillan (1987), Levin and Smith (1994) and Menezes and Monteiro (2000)). There are, however, two key differences. First, the auction literature typically focuses on the impact of entry on the seller's revenue and not social surplus. Second, since demand is downward sloping, the profit function of a firm under price competition is not linear in its price, whereas a bidder's profit in an auction is typically linear in prices, yielding different trade-offs.

2 Model

There are $n \geq 2$ potential producers of a homogeneous good. Each firm i, $i = 1, ..., n$, decides whether to enter the market for the homogeneous good and incur an entry cost equal to $k \geq 0$. Upon entry, firms choose their prices simultaneously with the knowledge of the number of firms that entered the market.

We denote by $C(q, c_i)$ firm i's cost function, where q is the quantity produced and c_i a firm-specific parameter. The cost function is continuous and twice differentiable, and increasing in both terms with $\frac{\partial^2 C(q,c_i)}{\partial q^2} \geq 0$. The firm-specific parameter c_i is private information, but it is common knowledge that each c_i is an independent draw from a continuous distribution function $F(.)$ with a support $[\underline{c}, \bar{c}]$, and density $f < \infty$. Each firm observes its firm-specific parameter before making their entry decisions.

The market demand is denoted by $D(p)$, which is also continuous and twice differentiable with $D'(p) < 0$. Bertrand competition means that if two or more firms enter the market, the firm that offers the lowest price captures the entire market. Given asymmetric information, and the focus on the strictly decreasing, symmetric equilibrium, the probability of a tie will be shown to be zero.

Using backward induction, we first characterize each firm's profit function upon entering the market. In particular, if there is only one firm that enters the market, its profit function is given by:

$$\pi_i^1(p_i, c_i) = [p_i q - C(q, c_i)]. \tag{1}$$

If more than one firm enters the market, then the expected profit of firm i, conditional on entry, becomes:

$$\pi_i^2(p_i, c_i) = [p_i q - C(q, c_i)] Pr(p_i < p_{j \neq i}), \tag{2}$$

where p_j is the lowest price among all other firms that entered the market.

Combining (1) and (2) allows us to express the *ex ante* expected profit of firm i as:

$$\Pi(p_i, k) = \max\{\rho_1 \pi_i^1 + \rho_2 \pi_i^2 - k, 0\}, \tag{3}$$

where ρ_1 is the probability that only firm i enters the market and ρ_2 is the probability that more than one firm, including firm i, enters the market. These probabilities will be computed later.

When firm i is the only firm that enters the market, it chooses the monopoly price and quantity, denoted by $p_m(c_i)$ and $q_m(c_i)$, to maximize profits. As explained above, when two or more firms enter the market, each firm sets a price as a function of its firm-specific parameter, and the firm with the lowest price captures the entire market. Below we characterize the symmetric Bayesian-Nash equilibrium, which consists of a decision for each firm of whether to enter, and a price setting strategy if the firm enters the market.

To characterize the equilibrium behavior of firms, we define $\tilde{c}$ as the threshold type which is indifferent between entering and not entering the market. As profits will be shown to be monotonic with respect to the firm-specific parameter, all firms with costs larger than $\tilde{c}$ are better off not entering and those with costs lower than $\tilde{c}$ enter the market.

Define $p^{-1}(p_i)$ as the type of firm i who chooses price p_i. We can then rewrite (2) as a function of $\tilde{c}$ as follows:

$$\pi_i^2(p_i, c_i, \tilde{c}, m) = [p_i q - C(q, c_i)] Pr(p^{-1}(p_i) < c_{-i} | c_{-i} < \tilde{c}), \tag{4}$$

where c_{-i} is the lowest type among other firms that also entered the market and m is the number of firms that entered.

Define $H(p_i, p_{j \neq i})$ as the probability that firm i's price is the lowest among all firms that entered the market. Focusing on a symmetric Bayesian-Nash equilibrium where firms choose their prices as a decreasing function of their types, the probability that firm i's price is the lowest among other entering firms is equivalent to the probability that its firm-specific parameter is the lowest among them. Differentiating (4) with respect to p results in the following first-order condition:

$$(D(p_i)+D'(p_i)p_i-D'(p_i)C'(D(p_i),c_i))H(p_i,p_{j\neq i})+(p_iD(p_i)-C(D(p_i),c_i))h(p_i,p_{j\neq i})=0, \tag{5}$$

where $h(p_i, p_{j\neq i})$ is the density of $H(.,.)$.

Proposition 1. *When more than one firm enters the market, there exists a symmetric price strategy $p^*(c_i)$, which is increasing in the firm-specific parameter and $p^*(c_i) < p_m(c_i)$.*

Having characterized the first-order condition that defines the symmetric equilibrium price conditional on entry, we now focus on the entry decision itself. To this end, we can rewrite firm i's *ex ante* expected profit as follows:

$$\begin{aligned}\Pi(c_i,\tilde{c},k) &= \rho_1\big[p_m(c_i)D(p_m)-C(D(p_m),c_i)\big] \\ &\quad+\rho_2\big[p^*(c_i)D(p^*)-C(D(p^*),c_i)(F(p_i(\tilde{c}))-F(p_i(c_i))^{n-1}\big]-k\end{aligned} \tag{6}$$

Given the threshold type $\tilde{c}$, we can compute ρ_1 and ρ_2. In particular, ρ_1 is the probability that all other types are higher than $\tilde{c}$, that is:

$$\rho_1 = (1-F(\tilde{c}))^{n-1}. \tag{7}$$

Furthermore, ρ_2 is the probability that at least one of the other firms' realized type is lower than $\tilde{c}$, which is given by:

$$\rho_2 = \sum_{m=1}^{n-1}\binom{n-1}{m}F(\tilde{c})^m(1-F(\tilde{c}))^{n-m-1} \tag{8}$$

We define $\bar{k}$ as the entry cost such that the monopolist with the lowest type is indifferent between entering the market, as the only firm, and not entering. As the next proposition shows, $\bar{k}$ can be thought of as an upper bound for the value of the entry cost that makes entry feasible.

Proposition 2. *For every $k < \bar{k}$ there exists a threshold type $\tilde{c}$ which is characterized by $\Pi(\tilde{c},\tilde{c},k) = 0$.*

3 Entry and social efficiency

As discussed, in the absence of entry costs, the issue of entry is moot. As Spulber (1995) shows, the lowest cost firm captures the entire market, and sets a price strictly below the monopoly price but larger than the competitive price. Moreover, Spulber (1995) also shows that increasing the number of firms leads to average cost pricing in the limit, approximating the competitive outcome.

When there are fixed costs of entry, however, there is a trade-off between more entry leading to more intense competition and the social costs in the form of the duplication of the entry costs. This section characterizes the level of entry that maximizes social surplus and shows that prices do not converge to the competitive equilibrium level when the number of firms increase.

Define $S(q_n)$ as the total surplus when $l \leq n$ firms enter the market. Formally, we can express $S(q_l)$ as follows:

$$S(q_l) = \int_0^{q_l} (p(t) - c_1(t, c))dt - lk, \tag{9}$$

where $q_l = D(p_l)$, which is the quantity produced at price p_l and c is the lowest realized cost among l firms.

Note that when only one firm enters, Equation (9) is equal to the monopolistic surplus evaluated at price p_m and quantity q_m. Our next result characterizes the number of firms that maximize total surplus. In the analysis that follows we address the integer constraint on the number of firms by rounding up the optimal entry level to the nearest integer number.

Proposition 3. *For any given cost realization of each firm, and for every entry cost $k < \bar{k}$, there exists an entry level l^* which represents the number of firms such that for any entry above l^*, the total surplus decreases. Also, the total surplus is increasing in l when entry level is below the l^*.*

The characterization in Proposition 3 is similar in spirit to the definition of socially optimal entry proposed by Cabral (2004). We note that l^* is socially optimal in a second-best sense. Even when l^* firms enter, the equilibrium price differs from the competitive equilibrium outcome that is, the equilibrium price is always larger than the cost of the marginal firm to enter the market as shown by the next Proposition.

Proposition 4. *In the presence of asymmetric information and entry costs, entry is almost always socially inefficient.*

Next we provide some comparative statics results that further our understanding of the impact of entry on total surplus.

Lemma 1. *When $k \to \bar{k}$, the optimal entry level goes to zero. When $k \to 0$, the optimal entry level goes to infinity.*

The above Lemma shows that the optimal level of entry could vary from zero to infinity as a function of entry costs. Intuitively, when the entry cost is large, the marginal effect of an additional firm entering the market becomes negative, which means that less entry is better. Conversely, when the entry cost is small, more entry implies getting closer to the competitive equilibrium price. In the limit, when the entry cost is zero, there is no negative effect on the surplus by adding another firm, and therefore, the optimal number of firms is infinite. The next proposition establishes the value of the entry costs that make the industry a natural monopoly in a normative sense.

Proposition 5. *There exists a $\hat{k} < \bar{k}$ such that for all entry costs above $\hat{k}$, it is optimal for only one firm to enter the market.*

It may well be that there are values of k greater than or equal to $\hat{k}$ for which more than one firm may enter the market, although it is optimal for only one firm to enter. That is, the industry may not be a natural monopoly in a positive sense. This case will be discussed in the example of the next section. In such cases, the standard regulatory approach is to restrict entry, and for the regulator to set price equal to average cost.

In the next proposition we consider the impact of an increase in the number of potential firms on the optimal entry level.

Proposition 6. *For any fixed positive entry cost k, the probability of having excess entry goes to one as the number of potential firms, n, goes to infinity.*

Proposition 6 is in sharp contrast to Spulber (1995) where an increase in the number of firms leads to an equilibrium price closer to the competitive equilibrium. This distinction arises of course from the existence of entry costs. In this case, the reduction in the equilibrium price decreases when n increases, whereas total entry costs born by society increase linearly in n.

4 Example

In this Section we provide an example to illustrate our main results. We assume that $n = 2$, and that the market demand is given by $Q = \alpha - \beta p$, where α, $\beta > 0$. Firm i's cost function, $i = 1, 2$, is equal to $C(q) = c_i q$. where c_i is the private information of firm i. Marginal costs are distributed uniformly on $[0, \bar{c}]$.

When both firms enter the market, they compete in prices, with the lowest price firm securing the entire market. As discussed in the previous section, we focus on strictly increasing, symmetric equilibrium, and therefore the probability of a tie is zero. If only one firm enters, then the single firm will choose the profit maximizing price.

Firm i's profit function when it is the only firm entering the market is given by:

$$\pi_i^1(p_i, c_i) = [p_i Q - C_i(Q)]. \tag{10}$$

The profit function when both firms enter the market is given by:

$$\pi_i^2(p_i, c_i, \tilde{c}) = [p_i Q - C_i(Q)] Pr(p_i < p_j | c_j < \tilde{c}). \tag{11}$$

Therefore, i's *ex ante* profit function is equal to:

$$\Pi(p_i, k) = \rho_1 \pi_i^1 + \rho_2 \pi_i^2 - k, \tag{12}$$

where, as above, ρ_1 and ρ_2 are the probabilities that either one or two firms enter the market, respectively. Note that as long as the *ex ante* expected profit is positive, firm i enters the market. Next we compute the optimal choices of the two firms under two scenarios.

First, suppose that only firm i enters. It chooses p_m to maximize (10), yielding the monopoly price. Note that π_i^1 has to be strictly positive at p_m, otherwise there is no entry. Second, assume that the two firms enter the market. As in the previous section, we look for a symmetric, monotonic equilibrium $\mathbf{p}^*$, where each firm i with cost c_i chooses a price $p^*(c_i)$. In this equilibrium, the probability of having the lowest price is equal to the probability of the lowest order statistic. So we can rewrite Equation (11) as follows:

$$\pi_i^2(p(c_i), k) = \big[(p(c_i) - c_i)(\alpha - \beta p(c_i))\big] \Bigg(F(\tilde{c}) - F(p^{-1}(p_i)) \Bigg). \tag{13}$$

Differentiating the above with respect to p yields the following first-order condition:

$$(\alpha - 2\beta p(c_i) + \beta c_i) \Bigg(F(\tilde{c}) - F(p^{-1}(p_i)) \Bigg) - \big[(p(c_i) - c_i)(\alpha - \beta p(c_i))\big] \frac{f(p^{-1}(p_i))}{p'^{-1}(p_i))} = 0 \tag{14}$$

Using symmetry, we obtain the following differential equation that characterizes the equilibrium:

$$p'(c) = \frac{(p(c) - c)(\alpha - \beta p(c))}{(\alpha - 2\beta p(c) + \beta c)} \frac{f(c)}{F(\tilde{c}) - F(c)} \tag{15}$$

For simplicity suppose $\tilde{c} = 1$. Then the above differential equation has the following solution:

$$p^*(c) = \frac{\alpha}{3\beta} + \frac{2c}{3}, \tag{16}$$

which is increasing in c as postulated.

It is straightforward to check the monopoly price is given by:

$$p^m(c) = \frac{\alpha}{2\beta} + \frac{c}{2}, \tag{17}$$

and therefore $p^m > p^*$. Also, the slope of p^* is bigger than the slope of p^m and they intersect at $\bar{c}$.

We can now use the equilibrium prices to compute ex-ante profits to assess the firms' entry decisions:

$$\Pi(c_i, k) = \rho_1\Big((p^m - c_i)(\alpha - \beta p^m)\Big) + \rho_2\Big((p^* - c_i)(\alpha - \beta p^*)\Big) - k \tag{18}$$

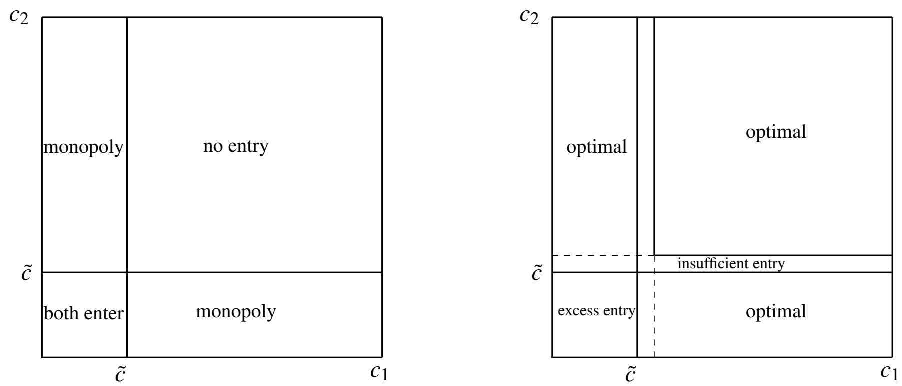
To further simplify the analysis, we assume that $\alpha = 2$, $\beta = \frac{1}{2}$, and $\bar{c} = 4 = \frac{\alpha}{\beta}$. We then set the marginal type at $\tilde{c} = 1$, and will determine the entry cost that makes this type the marginal type. To do so we compute the marginal type's ex ante profits:

$$\Pi(c, k|\tilde{c} = 1) = \frac{\bar{c} - 1}{\bar{c}}\Big((2 + \frac{c}{2} - c)(2 - 1 - \frac{c}{4})\Big) + \frac{1}{\bar{c}}\Big((\frac{4}{3} + \frac{2c}{3} - c)(2 - \frac{2}{3} - \frac{c}{3})\Big) - k \tag{19}$$

and set $\Pi(\tilde{c}, k|\tilde{c} = 1) = 0$ yielding:

$$\Pi(c, k|\tilde{c} = 1) = \frac{\bar{c} - 1}{\bar{c}}\Big((\frac{3}{2})(\frac{3}{4})\Big) + \frac{1}{\bar{c}}\Big((1)(1)\Big) - k = 0. \tag{20}$$

This gives an entry cost equal to $\frac{35}{32}$. For this entry costs, firms with realized costs less than one enter the market, and those with realized costs greater than one stay out. So all firms with realized costs less than one would enter the market.



(a) Entry (b) Efficiency

Figure 1: Entry and efficiency

We now focus on the optimal entry level. First, we note the change in price from one-firm entry to two-firms entry:

$$p^m(c) - p^*(c) = \frac{1}{6} - \frac{c}{6}. \tag{21}$$

The reduction in price from p^m to p^* yields an increase in quantity equal to $\Delta Q = \frac{1}{12} - \frac{c}{12}$. The overall change in the surplus is given by:

$$\Delta S = (\frac{1}{12} - \frac{c}{12})(\frac{4}{3} + \frac{2c}{3}) + \frac{1}{2}(\frac{1}{6} - \frac{c}{6})(\frac{1}{12} - \frac{c}{12}) - k. \tag{22}$$

We can check that the change in surplus is negative when two firms enter the market versus the case where only one enters. Figure 1(a) shows the number of firms entering given all possible realization of the costs. As shown when both costs are less than $\tilde{c}$, both firms enter the market and the final price is $p^*(c)$, set by the lowest cost among the two. In this case we have excessive entry, because the marginal surplus added by the extra entry is less than the entry cost. However, when there is only one firm with a realized cost lower than $\tilde{c}$, then the number of firms entering the market is optimal, that is, we cannot increase the surplus by adding or reducing a firm. Finally, when both costs are higher than $\tilde{c}$, then no firm enters the market. However, as long as the cost of at least one of the firms is below $c \approx 1.05$, then the entry of one firm would increase total surplus, and therefore we have insufficient entry in this case. Figure 1(b) depicts the possible cases in terms of efficiency.

We now investigate an increase in the number of firms and its impact on expected price and efficiency. First we note that the monopoly price does not change with n increases, and therefore we only need to focus on how the equilibrium price changes with n.

Suppose that $m \leq n$ firms entered the market. Then the probability of having the lowest cost among the entering firm becomes $(F(\tilde{c}) - F(p^{-1}(p_i))^{m-1}$. The differential equation in (23) would change as follows.

$$p'(c) = \frac{(m-1)(p(c) - c)(\alpha - \beta p(c))}{(\alpha - 2\beta p(c) + \beta c)} \frac{f(c)}{F(\tilde{c}) - F(c)}. \tag{23}$$

Given our example, the solution to the above for any m becomes,

$$p^*(c) = \frac{\alpha}{\beta(m+1)} + \frac{mc}{m+1}. \tag{24}$$

The above term decreases in m and at its limit the price approaches the marginal cost. In particular we have,

$$\frac{\partial p^*(c)}{\partial m} = \frac{c - \frac{\alpha}{\beta}}{(m+1)^2} < 0 \tag{25}$$

$$\frac{\partial^2 p^*(c)}{\partial m^2} = \frac{-2(c - \frac{\alpha}{\beta})}{(m+1)^3} > 0 \tag{26}$$

That is, when the number of producers increase, the expected price decreases. However, given that the maximum possible increase in the total surplus is lower than the entry cost, there is still excessive entry. In other words, the increase in the number of firms does not lead an improvement in overall efficiency.

5 Conclusion

This paper characterizes equilibrium behavior when firms have private information about their cost function to produce a homogeneous good, face entry costs, and compete in prices. We show that under asymmetric information and endogenous entry, there are different forces impacting on firm behavior and social efficiency.

First, only firms with costs below a threshold enter the market – so the number of competitors is lower than the total number of potential competitors. Second, firms that do enter price more aggressively than in the absence of entry costs – their pricing strategy reflects the fact that only firms with costs below the threshold will enter the market. Finally, upon multiple entry, the duplication of entry costs weighs negatively on total surplus. The optimal entry level, which maximizes expected total surplus, balances these opposing forces.

Our framework yields results that are more nuanced than those of the literature on free entry, where there is either insufficient or excessive entry. Depending on the distribution of costs, the value of the entry fee, and the number of potential entrants, it is possible to have both excessive and insufficient entry as parameters change.

We also show that the existence of entry costs fundamentally changes one of the key results of Spulber (1995) on the convergence of the equilibrium price to the competitive equilibrium. Instead, with entry costs, we have shown that the probability of excessive entry goes to one as the number of potential firms goes to infinity. This is perhaps an important reminder that competition regulators should not be tempted to focus on the number of potential competitors, rather than on the nature of competition.

Our results on the efficient level of entry also raise questions about relying on potential entry to constrain any market power of firms like Uber and Airbnb, which operate in markets where entry costs can be significant and there is a great deal of uncertainty about the nature of technology (and therefore costs).

6 Appendix

Proof of Proposition 1. First we calculate the probability that firm i's price is the lowest among others. In a symmetric, increasing equilibrium this probability is equal to the lowest order statistics conditional on all other types among those entered being lower than $\tilde{c}$. That is:

$$Pr(p^{-1}(p_i) < c_{-i} | c_{j \neq i} < \tilde{c}) = \big(F(\tilde{c}) - F(p^{-1}(p_i))\big)^{n-1} \tag{27}$$

We can then rewrite (5) as follows,

$$\begin{aligned}(D(p_i) + D'(p_i)p_i - D'(p_i)C'(D(p_i), c_i))\bigg(F(\tilde{c}) - F(c_i)\bigg)^{n-1} \\ - (n-1)(p_i D(p_i) - C(D(p_i), c_i))\frac{f(c_i)F((\tilde{c}) - F(c_i))^{n-2}}{p'(c_i)} = 0,\end{aligned} \tag{28}$$

Rearranging we obtain the following differential equation.

$$p'(c_i) = \frac{(n-1)(p_i D(p_i) - C(D(p_i), c_i))}{D(p_i) + D'(p_i)p_i - D'(p_i)C'(D(p_i), c_i)} \frac{f(c_i)}{F(\tilde{c}) - F(c_i)} \tag{29}$$

Note that $\pi^2(p^*(c), c) = p_i D(p_i) - C(D(p_i), c_i))$ and $\frac{\partial \pi^2(p^*(c), c)}{\partial p^*} = D(p_i) + D'(p_i)p_i - D'(p_i)C'(D(p_i), c_i)$. It follows from Spulber (1995) Proposition 2 that there is a unique increasing solution to the above differential equation with a boundary condition $\pi^2(p^*(\bar{c}), \bar{c}) = 0$.

We now show that $p^*(c_i) < p_m(c_i)$. Suppose $p^*(c_i) \geq p_m(c_i)$. Note that the monopoly price p^m is profit maximizing. Therefore, we must have,

$$D(p^m) + D'(p^m)p^m - D'(p^m)C'(D(p^m), c_i) = 0 \tag{30}$$

However, p^* cannot be equal to p^m because the first order condition in (5) is not satisfied. Also for $p^* > p^m$ the first term in (5) becomes negative while the second term is also strictly negative. That is, we must have $p^*(c_i) < p_m(c_i)$. □

Proof of Proposition 2. It is enough to show that for a given k, all the types below $\tilde{c}$ would enter the market and those with types above it would not enter. It is easy to check that the ex-ante profit function $\Pi(c, \tilde{c}, k)$ is strictly decreasing in c. Also $\Pi(0, \tilde{c}, k) > 0$ and $\Pi(\bar{c}, \tilde{c}, k) = -k$. Thus, due to continuity, there exists a unique $\tilde{c}$, such that $\Pi(\tilde{c}, \tilde{c}, k) = 0$. Also for all types below $\tilde{c}$ the ex-ante profit is positive and for all types above $\tilde{c}$ the ex-ante profit is negative.

□

Proof of Proposition 3. Differentiate (9) with respect to l yields:

$$\frac{\partial S(q_l)}{\partial l} = D'(p_l)p'(q_l)\Big(p(q_l) - c_1(p(q_l), c)\Big) - \int_0^{q_l} c_1'(t, c)\frac{dc}{dl}dt - k \tag{31}$$

The first terms of the RHS of (31) is positive, decreases with the number of firms that enter the market (l), and approaches zero when l goes to infinity.

When there are no entry costs, the surplus is maximized at the highest possible number of entrants. Also when the entry cost is $\bar{k}$, the surplus is zero. Therefore, with positive entry costs below $\bar{k}$, there exists a l^* such that for any entry above l^* the total surplus decreases and for entry below l^* it is increasing in l.

□

Proof of Proposition 4. Given that p^m is strictly greater than $p = mc$ it is enough to show that $p^* > mc$ for any given number of firms. Focusing on the case where more than one firms enter we can rewrite π^2 as follows:

$$\pi^2(p^*, c) = \int_c^{\hat{c}} c_2(D(p^*(x)), x)(1 - F(x))^{n-1} dx \tag{32}$$

Divide both sides by $(1 - F(c))^{n-1}$ and we have,

$$\frac{\pi^2(p^*, c)}{(1 - F(c))^{n-1}} = \int_c^{\hat{c}} c_2(D(p^*(x)), x) \frac{(1 - F(x))^{n-1}}{(1 - F(c))^{n-1}} dx \tag{33}$$

The fraction inside the integral goes to zero when n goes to infinity because it is less than one for $c < x$. Thus, it is enough to conclude that the profit goes to zero when n goes to infinity. It follow that while price may approach the lowest cost, it will be always larger than that cost for any finite number of firms entering the market. That is, the entry level will be socially inefficient. □

Proof of Lemma 1. When the entry cost is equal to $\bar{k}$, zero entry is optimal. In a close neighborhood, there is only a small subset of types that could make a positive profit from entry. Given that the payoffs are decreasing in the entry fee, it is straightforward to conclude that the lower the entry fee is the higher the number of potential entrants. On the other hand, when $k = 0$, then we have a situation similar to Spulber (1995), and then there is always a positive marginal effect on the overall surplus with an extra entry. So the optimal level of entry is infinity. On a close neighborhood of $k = 0$, the optimal number of entry is very large and again becomes smaller when k increases. □

Proof of Proposition 5. First note that by definition $\bar{k}$ the change in the surplus as shown in (31) will be negative at $\bar{k}$. In particular, we can denote $\bar{k}$ as,

$$\bar{k} = \int_0^{q^m} p(q) dq. \tag{34}$$

It is straightforward to check that for any extra entry the increase in the surplus by a reduction in price is less than $\bar{k}$ and therefore, the overall effect is negative. Also note that, when $k = 0$ then moving from one to two entry would increase the overall surplus. Given that (31) is continuous and decreasing in k, there exists a $\hat{k} < \bar{k}$ such that the (31) is zero when the entry changes from one to two. Therefore, for all entry costs above $\hat{k}$ the optimal level of entry is one.

□

Proof of Proposition 6. According to Proposition 3, for any given k, there exist an optimal entry level l^*. So we need to show that the probability that $m > l^*$ firms enter the market approaches one when $n \to \infty$. This probability is,

$$\sum_{m=l^*}^{n} \binom{n}{m} F(\tilde{c})^m (1 - F(\tilde{c}))^{n-m} \tag{35}$$

For a finite l^*, the above becomes a geometric series when n goes to infinity and has a limit equal to one.

□

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